Apparatus for Determining the Standard Deviation Mechanically.

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The calculating apparatus here described is a natural development from the "harp" harmonic analyser,* and similarly makes use of loaded strings, which apply forces to a flexible yoke, whose consequent movement resolves these forces automatically. The use of vertical strings in place of the horizontal ones employed in the "harp" being convenient for the special purpose of this apparatus, it became possible so to design it as to give quantitative amplitudes to the yoke movements, which in the "harp" were only qualitative.

Purpose of Employment.

The purpose of the apparatus is to enable tolerably precise calculations of standard deviation and other related statistical constants to be made quickly, and in the hands of persons other than trained computers. Much of the work done in any laboratory is necessarily restricted by considerations of time and labour to the production of figures which in any one set of observations are not sufficiently numerous to justify laborious statistical treatment. Or, equally, the known imperfections of material or of method may themselves result in errors in the computed statistical constant, so that computation of, *e.g.*, a standard deviation beyond two significant figures would have no real value. In both cases an approximate determination is all that the circumstances warrant, and only then if that determination can be made rapidly by anybody available. The present apparatus makes this possible.

General Construction.

The complete apparatus is shown in fig. 1. On a 6-foot vertical board are fastened the yoke-holder with its yoke (fig. 3) and the template-grid (fig. 2). From the yoke-holder (*yh*) the yoke (*y*) is suspended at either end by two short suspensions of fine cotton tape (typewriter ribbon). At the centre of the yoke there converges from below a set of twenty-five strings of sewing cotton (*s*); of these, the centre one (*sb*) is stronger than the rest and bears a "dead load" or "base" weight of 2000 grm. (*wb*). The other strings to right and left of this base string bear weights (*w*) proportionate in succession to

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* W. L. B., "A Simple Apparatus for Approximate Harmonic Analysis and for Periodicity Measurements," 'Roy. Soc. Proc.,' A, vol. 99 (1921).

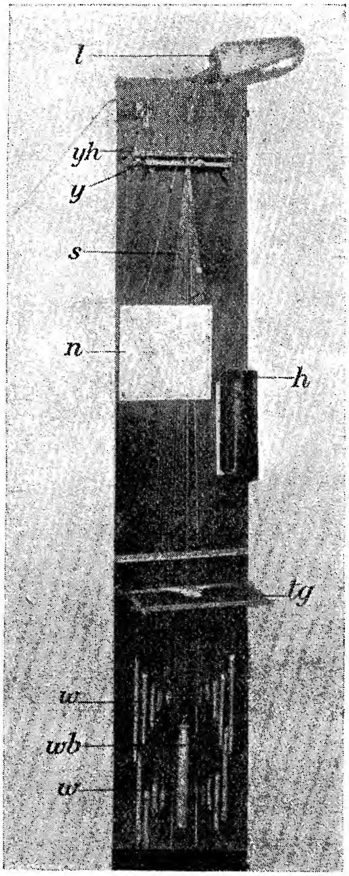

Fig. 1.—General view of standard deviation calculator.

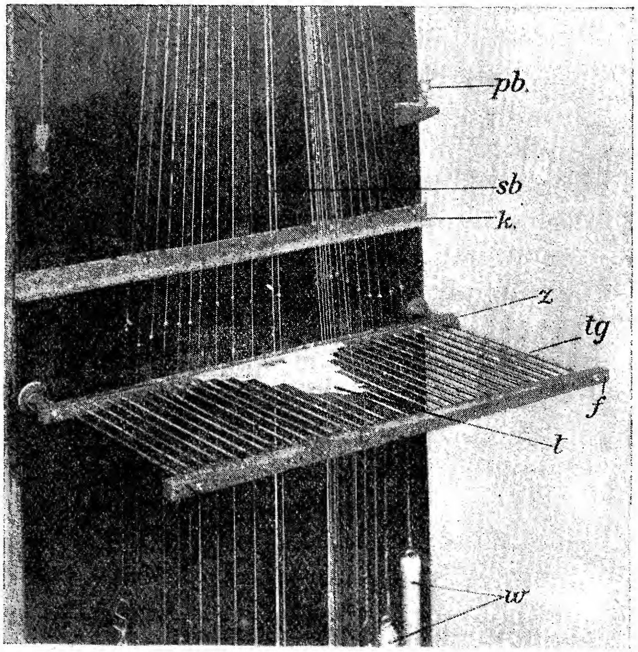

Fig. 2.—The template-grid, with a template in position.

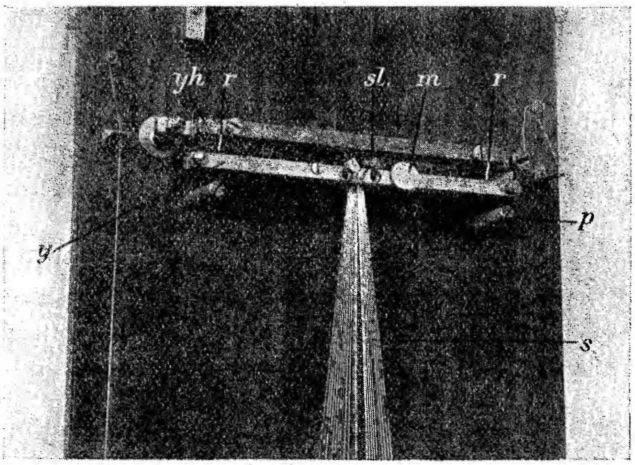

Fig. 3.—The yoke.

the square of their deviation from it. The actual weights, in grammes, on the strings are thus:—

144 100 64 36 16 4 (2000) 4 16 36 64 100 144
121 81 49 25 9 1 1 9 25 49 81 121

Each string is retained in its proper place in the series by passing through the template-grid (*tg*), below which the weights (of lead tubing) hang in two tiers, so as not to touch one another. The template-grid consists of two parallel brass bars (*f* and *z*), tied together by a rank of parallel steel rods, these being so arranged that at the grid every string is 1 cm. from its neighbours. The back of the grid is bolted to the base board, and the grid thus also serves as the operating table on which the template (*t*) under examination is laid, the back brass bar (*z*) defining the zero abscissa. The yoke-holder is set back slightly behind the grid, so that the weighted strings rest in contact with the rear brass bar of the grid at the zero for each ordinate of the frequency curve to be examined.

Method of Use.

To use the apparatus, the frequency curve in question is plotted on ordinary squared paper with ordinates 1 cm. apart and abscissæ 0·5 cm. apart. This is then cut out with scissors as a graph or template of rectangular co-ordinates, which is folded lengthwise and balanced on a middle point to find the mean ordinate. A straight-edge is then passed behind all the strings (*s*), excepting the zero string (*sb*), to pull them forward, and the template (*t*) is laid on the grid with its base against the back bar of brass (*z*), the rectangle which includes the mean ordinate being centred on the base string (*sb*). By withdrawing the straight-edge, the strings are allowed to fall back until each one rests on the edge of its appropriate rectangle in the template, or else on the brass bar as before (fig. 2).

The new force now exerted on the yoke in a plane perpendicular to that of the base board by any one string is proportionate, firstly, to the deflection, if any, of that string from its resting position by the template, *i.e.*, the height of its particular ordinate; and, secondly, if so deflected, to the square of the deviation of the position of that string along the grid from the base string, on account of the weight it carries.

The consequent movement of the yoke is read by a mirror (*m*) attached to it, as in the " harp," a lamp (*l*) fixed at the top of the base board, and a plane mirror (not shown) behind the observer, which reflects the image of the lamp filament upon a hooded scale (*h*) fixed on the base board conveniently for observation.

The movement there observed, which represents the "sum of the squares of the deviations," is quickly corrected for n, the number of points in the frequency, and the square root is also extracted by using a nomograph (n), fastened conveniently on the base board.

Allowing one minute for cutting the template, another minute suffices to complete the whole operation of determining a standard deviation; or $\sqrt{(\Sigma d^2/n)}$, irrespective of whether n is large or small. The result has not been corrected for the deviation of the true mean from the base ordinate chosen, since the correction is small if the standard deviation exceed some two units of the scale. For the true standard deviation σ_0 is related to the root mean-square deviation, δ, about an arbitrary origin by the equations

$$\delta_0{}^2 = \sigma_1{}^2 - e^2$$

where e is the difference between the mean and the arbitrary origin. That is, approximately, if e/δ be small:

$$\sigma_0 = \delta_1\left(1 - \tfrac{1}{2}\frac{e^2}{\delta_0{}^2}\right).$$

The maximum value of e in our case is $\frac{1}{2}$, so that the maximum value of the correcting term $\frac{1}{2}e^2/\delta^2$ is $1/8\,\delta^2$ if σ is 2, that is 1/32 or 3 per cent. If σ is 3 it is 1/72, or barely $1\frac{1}{2}$ per cent. When the data are being grouped the interval should be so chosen as to make the number of groups approach as nearly as is convenient to the limit for the instrument (25), in order to keep down possible error from this source.

*The Design of the Yoke.**

In the harp harmonic analyser we employed a light yoke of wire, since the load on its yoke was constant and no great accuracy was needed in measuring amplitudes, which, moreover, embraced only a very small angular movement In the new apparatus we habitually vary the total deflecting load on the yoke over a very wide range, equal sensitivity is required in all parts of the resulting scale deflection, and we further require these deflections to be strictly quantitative at all times. Zero constancy, however, as we had already seen in the harp, could take care of itself in these loaded string constructions. In the first model of the apparatus we used parallel strings, and proposed to obtain the correction for n automatically by inserting the template at proportionate distances from the yoke. This we found inadvisable; in any case, some form of nomograph or special scale would be required to extract the square root, and the operation $\Sigma d^2/n$ might as well be done in the same way: the whole base-board could then be simple. Further, the use of parallel strings subjects the yoke to a turning movement whenever the loading is not symmetrical round the base string; the constraining of the yoke against the rotation presented many difficulties. A bifilar suspension was useless, and on trying a four-fold suspension of the ballistic pendulum type, we met further difficulties.

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* Our use of the stretched-string construction may perhaps be interpreted as due to inadequate resources, or to lack of knowledge as to the possibilities of precise workmanship and sensitive design; but such is not the case. The resources afforded by the precision machine tools in the Mechanical Section of our Department were discarded deliberately—not on grounds of expense, but on grounds of efficiency, especially in respect of constancy.

It should be noted that any suspension material is extensible, that it is impossible to use mechanical connections (cup and vee bearings, etc.) since their frictional resistance may vary or may be excessive in such small angular movements, and, lastly, that construction which depends on accurate workmanship should only be admitted as a last resort. The objections to the ballistic pendulum suspension involved all three considerations; if long suspensions were employed, the method was accurate, but the angular rotation of the yoke, which we desire to measure, was trivial, while the bodily translation of the yoke from zero position brought in fresh corrections; short suspensions were thus inevitable, but these required great exactitude in superposing the upper and lower ends of the four main suspensions, and in the tension-equality of the cross strings. These conditions are impossible of permanent achievement, however much care may be taken in the original construction, since spun threads alone can give sufficient flexibility; the thinnest steel foil has sufficient stiffness to prevent quantitative reactions of the yoke at big deflections under the small loadings, unless impossibly heavy weights are used. The ballistic suspension was therefore abandoned.

We then returned to the divergent strings employed in the harp, again using an angle of divergence sufficiently small to have no appreciable error-effect, and even this can be removed by slightly increasing the weight carried by any one string so as to give the correct vertical component if necessary, but on the balance of minor considerations we consider it inadvisable to do so. A related, though separate, source of error might be expected in the friction of the splayed-out strings on their guides, as also on the template edge; actually, we find that this slight tendency to over-load or under-load any one string is effectively eliminated by vibrations during the setting up of a template and by the flexibility of the paper template itself. The yoke remained at full width, supported at its ends,* and carrying all the strings at its middle (fig. 3). Any tendency to rotation in a horizontal plane was thus reduced to vanishing point.

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* The black typewriter ribbons used for the suspension are scarcely visible in fig. 3. Their positions are indicated by *r r*.

In the early stages we had endeavoured to keep the weight of the yoke as low as possible, but this was obviously absurd, since in a weightless yoke the deflections produced by any weight on a string would be identical, and further the weighted strings themselves loaded the yoke. A base string (*sb*) was therefore added, on which the 2,000 grm. dead load weight (*sw*, fig. 1) hung permanently, thus defining the sensitivity of the apparatus, and rendering it practically independent of the displacements of the other weights, 1 to 144. The yoke could now be made heavy and strong in order permanently to assure two conditions: firstly, that it would never alter during use; and secondly, that one straight line should contain the flexion-points of all the strings, another straight line should contain the flexion-lines of both suspension ribbons, while these two lines should be in one plane and be fairly parallel.

These conditions were sufficiently satisfied by the only piece of machining employed in the apparatus. The yoke was made of two mild steel bars of $\frac{3}{8}$ inch square section, which were first clamped side by side and milled simultaneously along their upper faces. These milled faces were then opposed to one another, when any errors in the run of the milling cutter were naturally balanced out, the points of actual contact between the faces lying in one plane; and between them the two ribbons and twenty-five strings were simply clamped by bolts running through both bars (fig. 2, *y*).

As a minor precaution the yoke was provided in front with a small adjustable slider (*sl*), and after a 2 cm. concave mirror (*m*) had been fixed on it with wax and the yoke suspended in position, this slider was moved until the yoke balanced in the same position under its own weight as when it carried all the other weights also, totalling 3 kgrm.

It remained to support the yoke, and this was conveniently done by a similar pair of bars (*yh*) to clamp the upper ends of the ribbon suspensions (*rr*), these bars being mounted on two screwed rods projecting from the base-board so that the yoke could be moved in or out till the strings at rest made contact with the zero abscissa, *i.e.*, the back brass bar (*z*) on the template grid.

Two strong pegs (*p*), fixed just below the yoke, serve as safety stops to catch it in the event, not yet realised, of a ribbon suspension breakage. Two plumb bobs (*pb*) assist in fixing the board correctly. A rack (*k*) with slots cut in it is added to take the load of the weights, by means of lead shot clipped on each string, when the instrument is being moved about.

Errors of Observation.

These are insignificant, the chief ones residing in the actual reading of the position of the spot of light, and in reading the nomograph. Slight apparent changes of zero may result from warping of the lamp and plane mirror supports, or of the wax which secures the concave mirror to the yoke, but these are checked by a glance at the graduated scale before inserting a template. The friction of the strings on the steel rods of the template-grid might cause trouble if the latter were allowed to rust, but is not detectable if a film of vaseline is left on the steel. Internal friction in the typewriter ribbon and sewing cotton is also imperceptible and appreciably invariable.

Advantages of the Apparatus.

The apparatus may safely be set aside for months, with the certainty that it must function as well as ever when required. It gives standard deviations correct to within ±3 per cent. of the computed value, apart from the base correction, which itself is definitely limited to small dimensions by the method of use. The complete determination of a standard deviation can be made by any person, irrespective of statistical training, and with a marked saving of time. The time required for a determination is largely independent of the amount of calculation required, and the saving of time in comparison with ordinary calculation is thus greater with large arrays than with small ones. Both methods require identical procedure until the frequency curve has been plotted from the original data; after which the time required to obtain the final result was as follows, in three demonstration trials.

The following times were taken in stop-watch trials of the instrument against a practised computer. The computer employed a Burrough's Desk Adder, statistical tables and the slide-rule. Both the computer and the instrument started with the curve plotted, and the value of n known. Trial A demonstrated the superior dependability of the instrument in such hasty work.

(A) Curve with $n = 215$: Standard deviation $= 4{\cdot}98$.

(1) Computer's time, 6 minutes. Result entirely incorrect. (Repeated calculation.)

Computer's time, 8 minutes. Result correct.

(2) Instrument's time, 3 minutes 5 seconds. Result approximately correct (S.D. $= 4{\cdot}9$).

Of this trial the detailed timing was as follows:—

	Mins.	Secs.
Cutting template	1	50
Balancing for mean ordinate	—	15
Insertion and reading deflection	—	35
Removal	—	10
Reading nomograph........................	—	15
Total	3	5

(B) Curve with $n = 35$: Standard deviation $= 2{\cdot}18$.

This curve was purposely made of a shape inconvenient to manipulate in template form.

Both computer and instrument took 3 minutes with similar results. The detailed timing of the latter was as follows:—

	Mins.	Secs.
Cutting template	1	15
Balancing template	—	25
Insertion and reading deflection.........	—	45
Removal	—	10
Reading nomograph	—	15
	—	—
Total	2	50

(C) Curve with $n = 210$: Standard deviation $= 5{\cdot}68$.

Computer's time, 9 minutes.

Instrument's time, 3 minutes.

The floor space occupied is only 12″ × 18″.

The construction can be undertaken at any simply equipped work bench, as even the milling operation on the yoke bars can be replaced by lapping with carborundum.

Table of Test Results.

As a final test for the dependability of the apparatus, seven test templates were prepared, as shown in the following Table, of which Nos. 1 and 4 were particularly stringent tests.

It will be seen that the computed and observed results agree very closely over this wide range of test. It may also be pointed out that the height of the template grid permits of a maximum ordinate of twenty-seven ½ cm. units, and that the sensitivity of the apparatus can be doubled if no more than thirteen-unit ordinates are likely to be met with, since the templates can then be plotted in 1 cm. units.

Conclusions.

The paper describes and illustrates:—

(1) Another calculating apparatus derived from the harmonic analyser formerly described.*

(2) The present apparatus mechanically applies the method of least squares to calculating standard deviations of frequency distributions.

(3) By suitable modifications of the arrangement used in the harmonic analyser, the loaded-string system has been made to give quantitative results over a wide range of amplitudes and weightings.

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* W. L. B., *loc. cit.*, p. 286.

Seven Test Templates.

Deviation.	1.	2.	3.	4.	5.	6.	7.	Notes.
−12	—	4	—	—	—	—	—	No. 7 is the template shown in figs. 1 and 2.
−11	—	4						
−10	—	4	—	—	—	1		
− 9	—	4	—	—	1	2		
− 8	16	4	4	16	2	3		
− 7	16	4	4	14	2	1		
− 6	16	4	4	12	3	6		
− 5	16	4	4	10	3	5	1	
− 4	16	4	4	8	3	4	3	
− 3	16	4	5	6	4	10	7	
− 2	16	4	4	4	4	6	13	
− 1	16	4	4	2	4	8	17	
0	16	4	4	0	5	6	19	
1	16	4	4	2	5	6	17	
2	16	4	4	4	6	4	13	
3	16	4	4	6	4	6	7	
4	16	4	4	8	4	2	3	
5	16	4	4	10	3	—	1	
6	16	4	4	12	2			
7	16	4	4	14	1	2		
8	16	4	4	16	1	3		
9	—	4	—	—	1	5		
10	—	4	—	—	1	1		
11	—	4						
12	—	4						
N	272	100	68	144	59	81	101	
$\sqrt{(\Sigma d^2/n)}$								
(1) Computed ...	4·90	7·20	4·90	6·00	4·45	4·80	1·97	Computed, using 10-inch slide rule.
(2) Observed ...	5·0	7·2	4·9	6·3	4·4	4·8	2·0	Direct reading from apparatus.

(4) The apparatus is simple and inexpensive to construct, simple and quick in use, and retains its sensitivity and zero indefinitely.

My thanks are due to my assistant, H. Hancock, who constructed the finished apparatus shown in the photographs, besides helping me in various experiments on the preliminary model; and to G. Udny Yule, F.R.S., for revising the present account.

The work has been done in the Experimental Department of the Fine Cotton Spinners and Doublers Association, Ltd., at Rock Bank, Bollington, Macclesfield.

I am indebted to the Executive Directors of the Association for permission to publish this account.

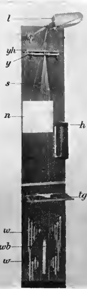

Fig. 1.—General view of standard deviation calculator.

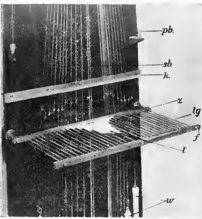

Fig. 2.—The template-grid, with a template in position.

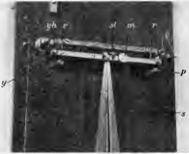

Fig. 3.—The yoke.